UW Math Circle October 8, 2015

1. You're back in the secret hall at Hogwarts, and Snape is standing outside three different doors. He tells you that Harry Potter is behind only one door and Voldemort is behind the other two doors, so choose carefully! He also tells you that the sign on Harry Potter's door is true and at least one of the other two signs is false.

I Voldemort is behind door II. II
Voldemort is in this room.

III Voldemort is in room I.

Where is Harry Potter?

2. The next time you're in the secret hall, Snape changes the rules on you. He tells you that Harry Potter is behind only one door, Voldemort is behind the other door, and the remaining room is empty. He also tells you that the sign on Harry Potter's door is true and the sign on Voldemort's door is false, while the sign on the empty room can be either true or false.

I Room III is empty. II Voldemort is in room I.

This room is empty.

Who is behind which door?

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- 1. Adam has 5 textbooks: one for biology, chemistry, Spanish, math, and history. He wants to arrange his books on a shelf in his room. How many ways can he do this if . . .
 - (a) he doesn't care what order they go in?
 - (b) he wants the math book to come first?
 - (c) he wants the math book to come second?
 - (d) he wants the first two books to be math and biology, but doesn't care what order they are in?
 - (e) he wants either the first or the last book to be his Spanish book?
- 2. How many ways are there to put a white rook and a black rook on a chessboard so they do not attack each other? What about a white king and a black king? What about a white knight and a black knight?



3. How many four digit numbers are there that begin with 1 and have exactly two identical digits? (Examples of such numbers are 1800, 1231, 1447.) What if the number doesn't have to begin with 1?

4. Sarah has one week's worth of dog treats: 2 bones, 2 biscuits, and 3 pieces of jerky. She gives her dog one treat each day. How many ways are there to do this?

5. How many positive integers less than or equal to 1000 do not have 2, 3, or 5 among their prime factors?