# UW Math Circle
November 6, 2014

1. Adam has 5 textbooks: one for biology, chemistry, Spanish, math, and history. He wants to arrange his books on a shelf in his room. How many ways can he do this if . . .

   (a) he doesn't care what order they go in?
   (b) he wants the math book to come first?
   (c) he wants the math book to come second?
   (d) he wants the first two books to be math and biology, but doesn't care what order they are in?
   (e) he wants either the first or the last book to be his Spanish book?

2. How many ways are there to put a white rook and a black rook on a chessboard so they do not attack each other? What about a white king and a black king?
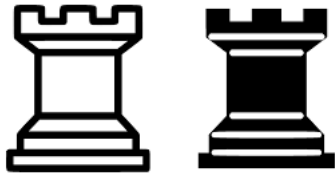
3. How many 10 digit numbers have at least two equal digits?

4. Sarah has one week's worth of dog treats: 2 bones, 2 biscuits, and 3 pieces of jerky. She gives her dog one treat each day. How many ways are there to do this?

5. We have 1000 numbers from 1 to 1000: 1, 2, 3, ..., 1000. Prove that if we pick 501 of them, one number we chose must be a factor of another. Would this still be true if we chose only 500 numbers?

6. How many diagonals are there in a regular n-gon? (A regular n-gon is a polygon with n sides whose sides and angles are all the same.)

7. Jane has exactly 24 songs on her playlist: 12 rock and 12 jazz songs. During her next run, she wants to listen to exactly 6 songs from each genre. Prove that no matter how the songs are shuffled, Jane will be able to find a block of 12 songs in a row, with exactly 6 from each genre.

8. Is it possible to fill a 4×4 grid with integers such that the sum of the numbers in each row is odd and the product of numbers in each column is odd?