

# UW Math Circle: Halloween Edition

October 30, 2014

Homework

1. Frankie had a super successful time trick-or-treating, and came home with 1000 pieces of candy. If he numbered them from 1 to 1000 and picked out 501 of them, prove that one number he chose must be a factor of another. Would this still be true if he chose only 500 numbers?



2. 7 rooks are placed on a  $7 \times 7$  chessboard, such that no rook attacks each other. Because it's Halloween, the rooks all simultaneously try to trick one another and each make a knight move. Could it still be that no rook attacks another?

3. A square pumpkin patch is divided into a  $10 \times 10$  grid of small square plots. Weeds have infested 9 of these plots. The weeds will spread to a plot if two of the plots adjacent to it (i.e. sharing a side) are already infested. Is it possible that the weeds will eventually take over the whole field?



4. Dracula only likes rock music and jazz music. On his playlist, he has 12 rock songs and 12 jazz songs. The next time that he flies around as a bat, he wants to listen to exactly 6 songs from each genre. Prove that no matter how the songs are shuffled, Dracula will be able to find a block of 12 songs in a row, with exactly 6 from each genre.