UW Math Circle

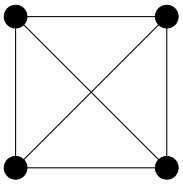
May 7, 2015

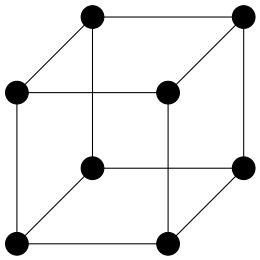
Remember, graphs are collections of dots (vertices) connected by lines (edges).

Definition: A graph is called **planar** if it can be drawn in such a way that none of its edges cross.

Definition: A graph is called **connected** if there is a path (possibly consisting of multiple edges) between every pair of vertices.

1. Are these graphs planar?





2. Draw four different planar graphs. Compute $V - E + F$, $3F$, and $2E$ in each case. What do you notice?

3. Which graphs with 3, 4, or 5 vertices are planar? Are there any of these graphs that don't seem to be planar?

4. Show that $3F \leq 2E$ for a connected planar graph.

5. Show that $3V - E \geq 6$ for a connected planar graph.

6. What does this tell you about your answer to problem 3?

7. There are three houses on a street. Each house needs to be hooked up to a source for three utilities – water, gas, and electricity. Is it possible to do this so that none of the supply lines cross?






8. Does problem 5 help you here? Show that if this graph is planar, then $4F \leq 2E$.