UW Math Circle

April 9, 2015

1. Suppose you have n pairs of parentheses and you would like to form valid groupings of them, where valid means that each open parenthesis has a matching closed parenthesis. For example, (()()) is valid, but ())()(is not. How many groupings are there for each value of n?

2. How many "mountain ranges" can you form with n /'s and n \'s , if all of the valleys must be above ground level?

   For instance,

   /\ is a mountain range for $n = 1$, and

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     /\
   /    \
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   is a mountain range for $n = 2$.

3. If $2n$ people are seated around a circular table, in how many ways can all of them be simultaneously shaking hands with another person at the table in such a way that none of the arms cross each other?