UW Math Circle

March 12, 2015

1. Each letter in Hagrid's name represents a different digit between 0 and 9. Show that

$$HAGRID \times H \times A \times G \times R \times I \times D$$

is divisible by 3.

2. At Middleton Middle School, there are 6 clubs: Drama Club, Math Team, Recycling Club, French Club, Spanish Club and Debate Team. The following students are members of each club:

Club	**Members**
Drama Club	Andrew, Bob, Catherine
Math Team	Andrew, Catherine, Dave
Recycling Club	Bob
French Club	Catherine, Dave, Elise
Spanish Club	Bob, Elise
Debate Team	Bob

What is the minimum number of days needed for the clubs to meet, provided that no two clubs with a shared member meet on the same day?

3. Tom and Jerry stole a chain of 7 sausages and are now trying to divide the bounty. They take turns biting the sausages at one of the connections. When one of them breaks a connection, he may eat any single sausages that may fall out. Tom takes the first bite. Each of them is trying his best to eat more sausages than his opponent. Who will succeed?

4. In how many different ways can you place 12 chips in the squares of a 4×4 chessboard so that

   (a) There is at most one chip in each square and

   (b) every row and every column contains exactly three chips?

5. Show that if n people attend a party, and shake hands with some other people attending the party (they cannot shake hands with themselves), then at least two people shook hands with the same number of people.

6. Is it possible to draw some number of diagonals in a convex hexagon so that every diagonal crosses EXACTLY three others in the interior of the hexagon? (Diagonals that touch at one of the corners of the hexagon don't count as crossing.)

7. Each cell of infinite graph paper contains one of the four numbers 1, 2, 3, or 4. Each of the four numbers is used somewhere at least once.

   Any cell has four neighbors . A cell is called *proper* if the number in that cell is equal to the number of different numbers that appear in one of its neighboring cells. In the picture on the left, the middle cell is proper because it contains the number 3 and its neighbors have three different labels (1,2, and 4). In the picture on the right, the middle cell is not proper because it contains the number 2, but its neighbors have four different labels.

	4	
2	3	2
	1	

	4	
2	2	3
	1	

   Question: Is it possible to fill the squares of an infinite piece of graph paper in such a way that every cell is proper?