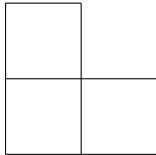# UW Math Circle
February 12, 2015

1. A tromino is an L-shaped piece, drawn below. Is it possible to cover an $8 \times 8$ chessboard with its upper left corner removed with trominoes? How about a $16 \times 16$ board with its upper left corner removed? $32 \times 32$? $2^n \times 2^n$?

2. You are playing the game of *'The Towers of Hanoi'*. It has three spindles on a base, with $n$ rings on one of them. The rings are arranged in order of their size - from largest on the bottom to smallest on the top. It is permitted to move the highest (smallest) ring on any spindle onto another spindle, except that you cannot put a larger ring on top of a smaller one. Prove that:

   (a) It is possible to move all the rings to one of the free spindles;

   (b) You can do so using $2^n - 1$ moves.

   (c) It is not possible to do so using fewer moves.

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1. Show that $1 + 3 + 5 + \cdots + (2n - 1) = n^2$.

2. Prove that the number $111\ldots111$ ($3^n$ 1's) is divisible by $3^n$.

3. Prove that the sum of degrees of the interior angles of an $n$-gon is $(n - 2)180$.
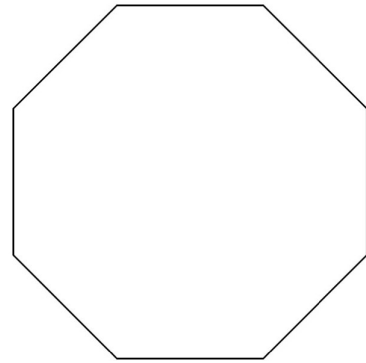
4. Prove that $7^n - 1$ is a multiple of 6 for all positive integers $n$.

5. Find a formula for the number of ways to cover a $2 \times n$ chessboard with dominoes (so that each square is covered, and no dominoes overlap). Prove your formula is correct.