UW Math Circle

February 12, 2015

Homework

1. Show that $1^2 + 2^2 + \cdots + n^2 = n(n+1)(2n+1)/6$

2. Show that if you have N lines drawn in the plane, then it is possible to color the regions between the lines black and white in such a way that no two neighboring regions are the same color.

3. Prove that for any positive integer n, $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.

4. We did this problem a few weeks ago:

   Suppose that a graph has n vertices, and that there is an edge between every pair of distinct vertices. How many edges are in the graph?

   We found the answer was $n(n-1)/2$ using different methods, but now see if you can prove that this is the correct answer using induction.