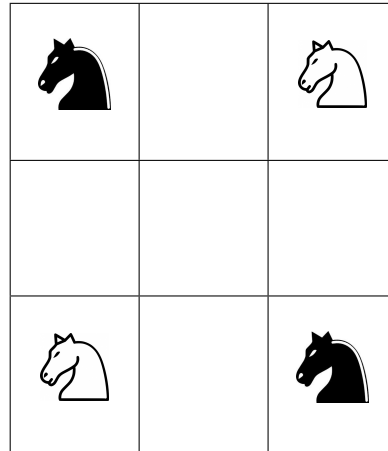
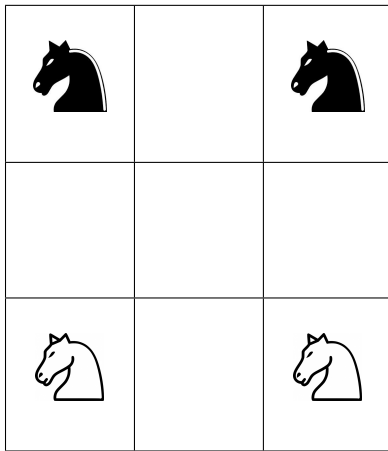
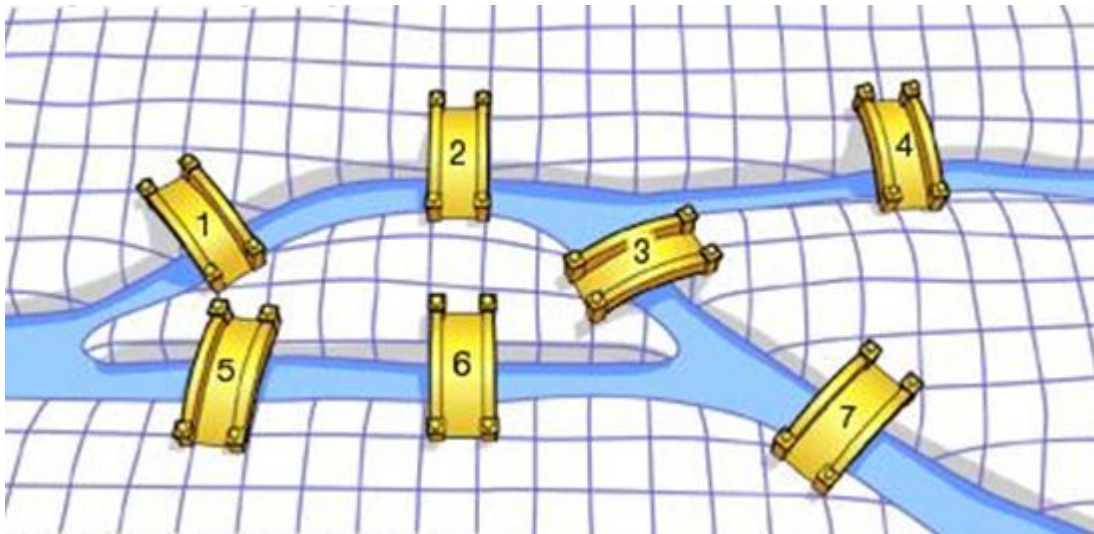


UW Math Circle
January 29, 2015

1. Due to some recent galactic advances, transporters allow you to make the following jumps between planets: Earth–Mercury, Pluto–Venus, Earth–Pluto, Pluto–Mercury, Mercury–Venus, Uranus–Neptune, Neptune–Saturn, Saturn–Jupiter, Jupiter–Mars, and Mars–Uranus. Is it possible to travel from Earth to Mars?
2. If you start with four knights placed on a chess board as shown on the left, is it possible to move them into the position on the right if two knights are never allowed to occupy the same square?



3. A long time ago, in the city of Königsberg, Lenny decided to take a walking tour of the city. Rivers divided the city into four land masses connected by 7 bridges. Was it be possible for Lenny to walk around Königsberg in such a way that he crossed each bridge exactly once?



4. Suppose that a graph has n vertices, and that there is an edge between every pair of distinct vertices. How many edges are in the graph?
5. In the faraway country of Septiland, there are 15 cities. Each city is connected to at least 7 other cities by a road. Show that it is possible to drive between any two cities, possibly passing through some other cities along the way.
6. In the neighboring land of Centropolis, 100 roads lead out of each city, and it is possible to travel along these roads from any city to any other. One of the roads is closed for repairs. Prove that it is still possible to get from any city to any other city.
7. Random Airlines flies to 21 cities, and from each city, they plan to have nonstop service to 7 others. (Nonstop service works in both directions, so that if the airline flies nonstop from A to B , then they also fly nonstop from B to A .) Prove that this flight schedule is impossible.

