UW Math Circle
January 15, 2015

1. Austin has a messy sock drawer full of red, blue, green, and yellow socks. If he's getting dressed in the dark, how many socks must he take from the drawer to guarantee that he gets a matching pair?

2. Sean works rescuing exotic birds. He has a total of 13 birds of 3 different colors: purple, blue, and green. Prove that he must have at least 5 birds of the same color.

3. If Sean rescued a 14th bird, would that guarantee he had 6 birds of the same color? If not, how many birds would he need to guarantee he had at least 6 of one color?

4. Sean decides to paint the floor of his very large birdhouse using two colors: white and pink. Show that no matter how he paints it, there must be two points on the floor of the same color exactly 1 foot apart.

5. No person has more than 500,000 hairs on his/her head. Prove that, in Seattle, at least two people have the same number of hairs on their head.

6. If you write the numbers 1, 2, ... , 10 around a circle (in any order), show that there must be 3 adjacent numbers whose sum is greater than or equal to 17.

7. Show that if 20 numbers are chosen from the sequence $1, 4, 7, 10, 13, \ldots, 94, 97, 100$, then there will be two of them that add up to 104.

8. What is the maximum number of bishops that can be placed on an 8×8 chessboard so that no two of them attack one another?