

UW Math Circle

January 15, 2015

Homework

The Pigeonhole Principle: If $N+1$ pigeons fly into N birdhouses, then some birdhouse will contain at least 2 pigeons.

The Generalized Pigeonhole Principle: If $k \cdot N + 1$ pigeons fly into N birdhouses, then some birdhouse will contain at least $k + 1$ pigeons.

1. Prove that among any three integers, there are two whose sum is even.
2. Jessica has a new backpack with 10 pockets in it, and she wants to put 44 things in it. Can she do this in such a way that each pocket has a different number of things?
3. Every square of a 2014×2014 chessboard is colored one of 2013 colors. In a given row or column, if two squares have the same color, then the entire row/column can be recolored that color. Is it possible to make the entire board one solid color?
4. Show that if you place 41 rooks on a 10×10 chessboard, then there will be at least five of them that do not attack one another.