Order of Battle

UW Math Circle

Session $\omega + 10$ (4 December 2014)

1. Prove or disprove: from any triangle you can cut out three congruent shapes each having area at least $\frac{1}{4}$ of the area of the triangle.
2. 100 spotlights of various shapes and sizes illuminate a square field with side length 1. The sum of the areas of these spotlights is greater than 99. Prove that there is a point in the field that is illuminated by all 100 spotlights.
3. Determine the fewest number of points that can be marked inside a convex n-gon so that any triangle with vertices in the vertices of the n-gon contains at least one of the marked points in its interior.
4. A $20 \times 20 \times 20$ cube is built out of 2000 $2 \times 2 \times 1$ bricks. Prove that you can stick a long needle somewhere through the cube (so that it comes out of the opposite side).
5. Find all pairs of integers x, y $(x \neq y)$ such that $x^y = y^x$.