

Problem Set 3

UW Math Circle

Session $\omega + 5$ (23 October 2014)

1. A connected graph with exactly one cycle is called a *pseudotree*.
 - (a) Show that a graph is a pseudotree if and only if it has as many vertices as edges.
 - (b) Conclude that a directed graph in which every vertex has out-degree 1 is a pseudotree.
 - (c) Use this to find another solution to the following problem, from Problem Set 3 (17 October 2013):

During a long budget debate in the United Galactic Senate, each Senator threw a space rock at exactly one other Senator. Prove that they can split into three parties in such a way that no one has thrown a space rock at anyone in his or her own party.

In fact, show that you can do this so that one of the parties contains at most one United Galactic Senator.
2. (Jordan's theorem¹) The *center* of a graph is a vertex v for which the largest distance from v to some other vertex is minimal.

Prove that a tree either has one center or has two centers that are connected by an edge.
3. A graph with no cycles is called a *forest* (because each of its components is a tree).

What are there more of: forests with 10 (labeled) vertices or trees with 11 (labeled) vertices?



"We found her wandering at the edge of the forest. She was raised by scientists."

¹If you know some French, you may find Jordan's original article, in a German journal, interesting: Camille Jordan, "Sur les assemblages de lignes", Journal für die reine und angewandte Mathematik, 1869. You can read it by following the link at the Math Circle website.