Problem Set 14

UW Math Circle

Session $\omega + 25$ (7 May 2015)

1. Prove that a parallelogram cannot be cut into an odd number of triangles of equal area.
2. Show that the area of a polygon in the plane with vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$ is

$$\left| \frac{1}{2} \left((x_1 y_2 + x_2 y_3 + \cdots + x_{n-1} y_n + x_n y_1) - (x_2 y_1 + x_3 y_2 + \cdots + x_n y_{n-1} + x_1 y_n) \right) \right|.$$

(If you played "math contests", you may know this as the *shoelace formula*.)

3. (Moscow City 1992) Prove that in any centrally symmetric convex polygon one can inscribe a rhombus at least half the area of the polygon.

4. (MHO 2012) Katniss is thinking of a positive integer less than 100: call it x. Peeta is allowed to pick any two positive integers N and M, both less than 100, and Katniss will give him the greatest common divisor of $x + M$ and N. Peeta can do this up to seven times, after which he must name Katniss' number x, or he will die. Can Peeta ensure his survival?