Problem Set 13

UW Math Circle

Session $\omega + 24$ (30 April 2015)

1. In a certain country on the plane, despite many obstacles – lakes, oceans, swamps – any point can be reached by walking in a straight line both from the capital and from the largest steel factory. Show that any point can be reached by walking in a straight line from any point along the straight railroad from the capital to the steel factory.

   (That is, prove that if a shape is star-shaped with respect to two points A and B, then it is star-shaped with respect to every point on the segment from A to B. Also, same question for great circles on the sphere.)

2. A convex polygon A is contained within a convex polygon B. Show the perimeter of A is not greater than the perimeter of B. (Hint: By induction on the number of sides of A which are not contained in sides of B.)

3. Determine the number of ways for n people and their n worst enemies to stand in a line so that nobody stands next to their worst enemy. (Same question for sitting around a table. If sums scare you, try $n = 4, 5, 6$.)

WHY MATHEMATICIANS ARE NO LONGER INVITED TO PUBLIC FORUMS