Problem Set 11

UW Math Circle

Session $\omega + 22$ (16 April 2015)

1. (a) How many graphs are there on n labeled vertices?
   (b) How many graphs are there on 5 labeled vertices with no isolated (degree 0) vertices? (If you are scared of sums, set $n = 5, 6, 7$.)
2. Ten insurance salesmen invade a restaurant where five couples are dining. Determine the number of ways for the salesmen to distribute themselves among the five tables so that there is at least one salesman at each table.
3. Biologists gather genetic data from 9 populations and compute, for each pair, a number expressing the amount by which their genomes differ.

	A	B	C	D	E	F	G	H	I
A		22	45	33	31	38	38	43	41
B			21	35	35	31	32	46	43
C				40	35	32	33	41	46
D					37	35	19	38	30
E						21	23	29	25
F							22	27	20
G								21	15
H									16

   Interpret these numbers as graph edge lengths, and find a possible taxonomic classification by finding a minimal spanning tree and choosing a root.

4. The town of Bordaville is holding a mayoral election, and 17 candidates enter the race. To decide on a winner, each voter ranks the candidates from best to worst (without ties). Each candidate gets 16 points for every voter who ranked her first, 15 points for ever voter who ranked her second, 14 points for every voter who ranked her third, and so on, ending with 1 point for every voter who ranked her second-to-last and no points for the voters who rank her last. The candidate with the maximum number of points wins and everyone else loses.

   If a candidate is ranked #17 by more than half of the voters, is it possible for that candidate to win?