Problem Set 7

UW Math Circle

Session $\omega + 12$ (15 January 2015)

1. (Transylvanian lottery problem) In a certain lottery, you must select three out of the numbers $1, 2, \ldots, 14$ on a lottery ticket. The lottery organizers will randomly select three winning numbers. You win if you selected at least two of the three winning numbers. Determine the least number of tickets you must buy, and what numbers to select on them, to be sure of winning on at least one of the tickets. (Hint: Use the Fano plane.)

2. The New England Internationalists and Seattle Cormorants (9 players each) played each other in the 2015 HyperBole. Is it possible that over the course of the game, each player tackled or was tackled by exactly three players on the opposing team and every two players tackled or were tackled by at most one player in common? (Hint: Interpret as a projective configuration.)

3. Prove that if a nonconstant increasing arithmetic sequence of positive integers contains a perfect square, then it contains infinitely many perfect squares.

4. Prove that there is no equilateral triangle with its vertices at points in the plane with integer coordinates.