Things to Think About 2

UW Math Circle, Winter 2013

This week, we talked about different ways of tallying votes in an election with more than two people. For example, consider the following set of voter preferences, in an election between Avon, Beverly, Chelsea, Dennis, and Everett:

Number of voters	18	12	10	9	4	2
First choice	Avon	Beverly	Chelsea	Dennis	Everett	Everett
Second choice	Dennis	Everett	Beverly	Chelsea	Beverly	Chelsea
Third choice	Everett	Dennis	Everett	Everett	Dennis	Dennis
Fourth choice	Chelsea	Chelsea	Dennis	Beverly	Chelsea	Beverly
Fifth choice	Beverly	Avon	Avon	Avon	Avon	Avon

That is, 18 people prefer Avon over Dennis over Everett over Chelsea over Beverly; 12 people prefer Beverly over Everett over Dennis over Chelsea over Avon; and so on.

Here are some things to think about.

1. Try using the methods we came up with in class on the above set of voter preferences. Who wins, under each method?

2. Are there any candidates who never win? If so: aww, that's sad. Try to come up with a reasonable way of counting the votes so that they're the winner.

3. A **one-on-one winner** is a candidate who would beat *any* other candidate if it were just a one-on-one race between them. For each of the methods we've seen (or that you just made up): if there's a one-on-one winner in a given election, does he or she necessarily win the whole election?

4. A **one-on-one loser** is a candidate who would *lose* to any other candidate if it were just a one-on-one race. Can any of the methods we've discussed ever elect a one-on-one loser?

5. A candidate is **meh** if zero of the voters consider him or her to be the best candidate. Is it possible for a meh candidate to win under any of the systems we've discussed? Which ones?

6. A voting system is **backwards** if moving a candidate up in a list (while keeping everything else relatively the same) might cause them to go from being the winner to being a loser. Which of the methods we've discussed are backwards? If a method is backwards, give an example where this can occur. If it's not backwards, prove it!