Things to Think About 1

UW Math Circle, Winter 2013

In class we discussed “figures” - collections of “nodes” that are connected pairwise by “pathways.” We call a figure “flattened” when we can draw it on a piece of paper so that none of the pathways cross.

1. We proved and postulated many interesting things about figures, including a famous result known as *Euler’s Formula*, which states that in a flattened figure the number of nodes (N) plus the number of regions (R) equals two plus the number of pathways (P): N+R=P+2. Our proof of Euler’s Formula was a little bit hand-wavy: let’s try to make it a bit more rigorous. Think about taking some arbitrary figure and steadily decreasing the number of regions by cleverly removing pathways. Can you find an invariant - something that is conserved every time you remove an edge? Can you show that Euler’s Formula holds for figures with only one region, and then show that we can remove pathways from any figure to get to a figure with only one region?

2. We found that some figures that we could not flatten on paper (or on the chalkboard) we could flatten if we connected the left edge of the board to the right edge (so that a pathway traveling off the right edge would reappear on the left) and connected the top edge of the board to the bottom. We found that our N+R=P+2 formula does not hold for figures drawn such a board, and in class we hypothesized that a different but similar formula holds instead: N+R=P. Try to rationalize or prove this.