UW Math Circle, Autumn 2013 - Homework 7

Due November 21, 2013

This week we extended our counting approach from last week to determine the number of ways to tile a ring of n tiles with 1×1 and 1×2 tiles (see problem 3 of homework 6). We denoted this number by L_n and found that

$$L_1 = 1 \qquad L_2 = 3 \qquad L_n = L_{n-1} + L_{n-2}$$

Notice that the L_n satisfy the same recurrence relation as the Fibonacci numbers f_n, but start with $1, 3$ rather than $1, 1$. We used our interpretations of f_n and L_n as the number of ways to tile a $1 \times n$ board and ring, respectively, to prove some interesting relationships between f_n and L_n. See the weekly email for details.

Practice your understanding of our discussion of the f_n and L_n numbers by solving the following problems. **Please write down your solution to at least one of these problems and turn it in next week.**

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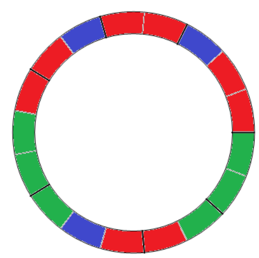
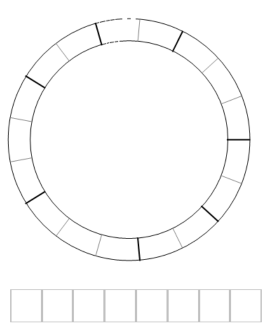
1. In class we showed that the number of ways to tile a ring of n tiles with 1×1 and 1×2 (curved) blocks is L_n. Find the number of ways to tile a ring of n tiles with 1×1 and 1×2 and 1×3 (curved) blocks. Give this sequence a name! (Be creative!)



2. In class we showed that $f_n^2 = f_{n-1}f_{n+1} + (-1)^n$. Show that $L_n^2 = L_{n-1}L_{n+1} + 5 \cdot (-1)^n$



3. In class we showed that $f_{2n-1} = L_n f_{n-1}$. For what n, m, and N is it true that $f_N = L_n f_m$?

4. Show that

$$f_n + f_{n-1} + f_{n-2} + 2f_{n-3} + \ldots + 2^{n-2} f_0 = 2^n$$