UW Math Circle, Autumn 2013 - Homework 6

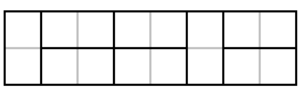
Due November 14, 2013

For the next few weeks we will be learning how to count things. This week we started by counting the number of ways to write a number n as a sum of 1s and 2s. Interestingly, we found that the number of ways to do this is the n^{th} Fibonacci number. We then showed that this problem is identical to determining how many ways there are to tile a $1 \times n$ board with 1×1 and 1×2 blocks, meaning that they share the same answer: the Fibonacci numbers. We then thought about how our answer would change if we were tiling a *loop* of n squares, but we'll talk about that in much more length soon.

For reference, we defined the Fibonacci numbers to be:

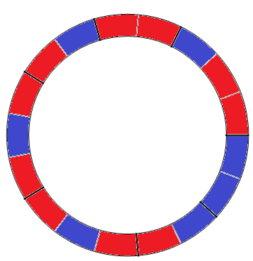
$$f_0 = f_1 = 1 \qquad\qquad f_n = f_{n-1} + f_{n-2}$$

The first few Fibonacci numbers are $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ Practice your understanding of our discussion of the Fibonacci numbers by solving the following problems. **Please write down your solution to one of these problems and turn it in next week.**



1. Show that the number of ways to tile a $2 \times n$ board with 1×2 dominoes is the n^{th} Fibonacci number f_n. Find the number of ways to tile a $3 \times n$ board with 1×3 shapes. Give this sequence a name! (Be creative!)

2. In class we showed that the number of ways to tile a $1 \times n$ board with 1×1 and 1×2 tiles is the n^{th} Fibonacci number f_n. How many ways are there to tile a $1 \times n$ board with 1×1, 1×2, and 1×3 tiles? What if you also allow 1×4 tiles? Give these sequences names!



3. How many ways are there to tile a *ring* of n tiles with 1×1 and 1×2 (slightly curved) tiles? Give this sequence a name! (The picture to the left is an example of one way to do this with $n = 17$. In the picture, blue tiles are 1×1 and red tiles are 1×2.)

4. Use our definition of the Fibonacci numbers - the number of ways to tile a $1 \times n$ board using 1×1 and 1×2 tiles - to show that

$$f_0 + f_1 + f_2 + \ldots + f_n = f_{n+2} - 1$$