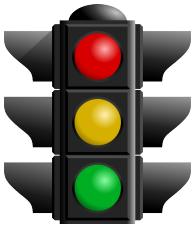
UW Math Circle, Autumn 2013 - Homework 5

Due November 7, 2013

This week we continued our discussion of choice machines and saw how they give rise to machine *memory*. See the weekly email for details.

Practice your understanding of choice machines and machine memory by working through the following problems. Write up some of your solutions!



1. Alex has a very boring job: he's in charge of switching the light at a traffic stop from red to green to yellow to red etc. To get off work early, he decides to build a machine that does his job for him. Help Alex create a new kind of machine that accepts only words consisting of the letter 'a' (where each 'a' represents a change of light color) and has *three* output nodes (one for each possible color).

2. While Jennifer was telling the story of the Sphinx at Thebes, she misremebered the riddle and told this one instead: "Design a machine that accepts words consisting of 'a', 'b', and 'c' and say YES only to words such that all 'a's are followed by 'bcb' or 'ccb'. " Help the hero of Jennifer's story (Oedipus) solve this riddle.

3. Design a machine that accepts words consisting of '1's and '0's and says YES if the word contains the substring '1100' *or* does not contain the substring '1010'

4. *Challenge*: you have two machines M_1 and M_2 that both accept words consisting of '0's and '1's, but say YES to different collections of words S_1 and S_2. Suppose that M_1 has Q_1 nodes and M_2 has Q_2 nodes. In terms of the sizes of Q_1 and Q_2, can you find an upper bound on the length of the smallest word that yields different answers from M_1 and M_2?