UW Math Circle, Autumn 2013 - Homework 4

Due October 24, 2013

This week we continued to explore which combinations of acceptable word sets produce another acceptable word set. Suppose S_1 and S_2 are two word sets for which we can design machines. Over our last two meetings we have designed machines that accept:

- the set of words that are in S_1 *or* are in S_2
- the set of words that are in S_1 *and* are in S_2
- the set of words that are in S_1 but not in S_2
- the set of words that result from reversing every word in S_1
- the set of all words not in S_1
- the set of words that result from concatenating finitely many words in S_1

Practice your understanding of our in-class constructions by solving the following problems.

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1. Kristen has machines $M_1, M_2, M_3, \ldots$ that say "yes" to the word sets $S_1, S_2, S_3, \ldots$, respectively. Is it possible for her to design one machine that says "yes" to all of the words in each of the S_i's? Is it possible for her to design one machine that says "yes" to only the words that are common to all of the S_i's?

2. Alex has a machine that says "yes" to the word set S. Julia chooses a subset of the words in S and calls it S'. Is it possible for Alex to design a machine that says "yes" only to words in S'?

3. Jonah has a machine that says "yes" to some word W. He notices that W visits some node N more than once. Prove that there are infinitely many other words that Jonah's machine must accept.

4. Professor McGonagall has a machine that says "yes" to every word in the set S. Help her design a machine that says "yes" only to words that are the first two-thirds of a word in S.