UW Math Circle, Autumn 2013 - Homework 3

Due October 17, 2013

This week we talked more about the word-acceptance machines from last week, and got a glimpse of some sets of words for which we *cannot* build such a machine.

Try practicing your understanding of these machines by solving the problems below.

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1. Create a machine whose input is any word consisting of the letters 'a' and/or 'b', which says "yes" to a word if the number of 'a's is odd or the number of 'b's is even, but says "no" if both of those things are true. Make your machine as simple as possible.

2. Make a machine that accepts a word if and only if it contains the word "cat" inside it.

3. Can you make a machine whose input is any string of digits in base ten, which says "yes" if and only if the resulting number is a power of 2? If so, construct it. If not, prove that it is impossible.

4. In the card game Crazy Eights (which uses a standard 52-card deck), players take turns playing cards on top of a pile in the middle of the table, with the rule that each card must match either the suit or rank of the card before it. Furthermore, eights are "wild", and the player who plays an eight may declare its suit to be anything.

Consider the set of "words" whose letters are sequences of playing cards in a 52-card deck. Describe how to make a machine that accepts a word if and only if it is a valid sequence of cards that could be played in Crazy Eights. (You don't have to actually draw the machine. Just describe how you *would* draw it.)

5. The *concatenation* of two words is what you get when you write them back-to-back. For example, the concatenation of `car` and `rot` is `carrot`. Suppose you have a machine that accepts the word set S_1 and a machine that accepts the word set S_2 (over the same alphabet). Can you make a new machine that accepts only concatenations of words in S_1 with words in S_2?