UW Math Circle, Autumn 2013 - Homework 2

Due October 10, 2013

This week we learned about a certain kind of machine, which takes as input a (possibly nonsense) word, and yields as output either a "yes" or a "no". The machine consists of a network of nodes (one of which is marked as the start node, and some of which are labeled as "yes" nodes) connected by arrows (which are labeled by letters). We use this machine by beginning at the start node and following, in order, the arrows corresponding to the letters of the word. Once we're done following all the letters of the word, we get a 'yes' from the machine if and only if the node we end on says so. In this way, the machines allow us to tell whether or not the word follows a certain rule.

Try practicing your understanding of these machines by solving the problems below.

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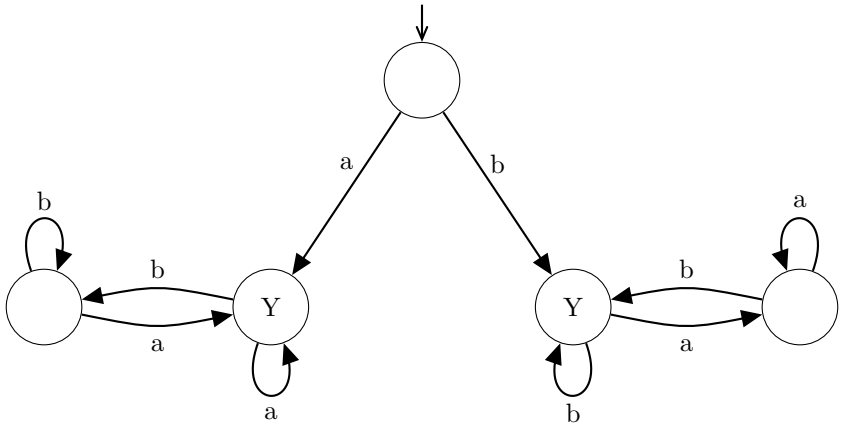
1. Consider the machine drawn below, which takes as input any word using the letters 'a' and 'b' and starts at the top node. Describe, as simply as you can, which words this machine says 'yes' to.



2. Create a machine whose input is any word consisting of the letters 'a', 'b' and/or 'c', and which says 'yes' to a word only if it starts with a 'c' and ends with a 'b'.

3. Create a machine whose input is any word consisting of the letters 'a' and/or 'b', and which says "yes" to a word if and only if the number of 'a's is a multiple of four and the number of 'b's is a multiple of three.

4. Create a machine whose input is any word consisting of the digits '0' and/or '1', and which says "yes" to a word if and only if, when read in binary, it is a number which is a multiple of seven.

5. Suppose we have a machine which says 'yes' to a certain set of words. Show how you can modify this machine to only say 'yes' to words that are accepted by the original version *and* have an even number of letters.