UW Math Circle, Autumn 2013 - Homework 1

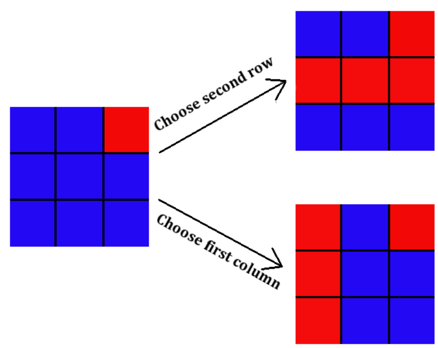
Due October 2, 2013

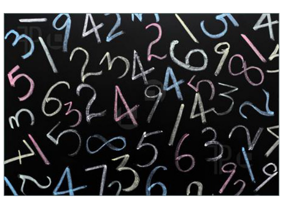
Welcome back to the UW Math Circle! This week we reviewed how to solve problems using *parity* (whether a number is even or odd) and then re-visited the broader subject of *invariants*. Invariants are a property of a given problem that the "rules" of the problem never change - see the weekly email for some of the examples we went over in class. Try practicing your understanding of invariants by solving the problems below.

---

1. A chess king traverses an 8×8 chessboard. He visits every square exactly once before returning to his original square. Prove that the king made an even number of diagonal moves.

2. A 3×3 board has every square colored blue except for the top right square, which is red. Jonah is allowed to choose any row or any column and reverse the colors of every square in his row/column. Can Jonah ever make the entire board red?





3. The numbers $1, 2, 3, \ldots, 2013$ are written on a blackboard. Alex is allowed to erase any two numbers and replace them them with their (non-negative) difference. Is it possible for Alex to, after many operations, make every number on the board a 0?

4. Farmer Steve has a 10 acre $\times$ 10 acre field. Unfortunately, nine of his 1 acre $\times$ 1 acre plots have become infested with weeds. The weeds can spread to any other 1 acre $\times$ 1 acre plot that has *at least* two weed-infested plots adjacent to it (diagonals don't count!). Prove that no matter how the original nine plots were distributed the weeds will never be able to take over all of Steve's territory.