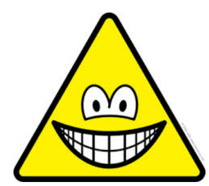
UW Math Circle

February 20, 2014

1. Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 9.

2. Is it possible to rearrange the digits of the number 123456 to get a number that is divisible by 11?

3. Suppose you have a triangle in which all three side lengths and all three heights are integers. Prove that if these six lengths are all different numbers, then it is not possible for four of the six numbers to be prime.



4. Show that all of the numbers in the sequence 10017, 100117, 1001117, 10011117... are divisible by 53.

5. (a) Show that for any prime number p that is not 2 or 5, there exists a power of p whose last two digits are 01.

   (b) show that for any M, there is a power of p whose last M digits are 00...01 ($M-1$ zeros followed by a 1).

6. Fred and George have designed the Amazing Maze, a 5×5 grid of rooms, with Adorable Doors in each wall between rooms. If you pass through a door in one direction, you gain a gold coin. If you pass through the same door in the opposite direction, you lose a gold coin. The brothers designed the maze so that if you ever come back to the room in which you started, you will find that your money has not changed. Ron entered the northwest corner of the maze with no money. After walking through the maze for a while, he had 8 shiny gold coins in his pocket, at which point he magically teleported himself out of the maze. Knowing this, can you determine whether you will gain or lose a coin when you leave the central room through the north door?