# UW Math Circle
February 6, 2014

1. Use the Euclidean Algorithm to find $\gcd(2n + 13, n + 7)$ for any whole number n.

2. You are given two empty jugs of water – one of them holds can hold exactly 10 gallons of water, and the other can hold exactly 38 gallons of water. You can pour water between the jugs, empty the jugs into the ocean, or refill them from the ocean. Which quantities of water can you measure in this way?

3. Suppose you have a 3×3 grid filled with positive numbers such that

   - the product of the numbers in each row is 1
   - the product of the numbers in each column is 1
   - the product of the numbers in each 2×2 square is 2

   What number is in the center of the square?
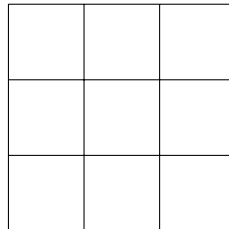
4. (a) Show that for any prime number p that is not 2 or 5, there exists a power of p whose last two digits are 01.

(b) show that for any M, there is a power of p whose last M digits are 00...01 ($M - 1$ zeros followed by a 1).

5. Show that all of the numbers in the sequence 10017, 100117, 1001117, 10011117... are divisible by 53.

6. Find gcd(A,B), where A $=$ 111111....1 with 100 digits and B $=$ 1111....1 with 60 digits.

7. What is the last digit of $1^{2012} + 2^{2012} + 3^{2012} + 4^{2012} + 5^{2012}$?
What is the last digit of $1^{2012} + 2^{2012} + ... + 2012^{2012}$?