UW Math Circle

January 16, 2014

1. What do you notice if you make the following reductions:
   - $17 \equiv$ _______ mod 3
   - $19 \equiv$ _______ mod 3
   - $17 + 19 \equiv$ _______ mod 3
   - $2 + 1 \equiv$ _______ mod 3
   - $17 \cdot 19 \equiv$ _______ mod 3
   - $2 \cdot 1 \equiv$ _______ mod 3
2. Show that $n^3 + 2n$ is divisible by 3 for any integer n.
3. In the faraway land of Moneytown, there are coins corresponding to each dollar amount: they have a \$1 coin, a \$2 coin, a \$3 coin, and so on. If a resident of Moneytown has $n+1$ coins, show that she has two coins whose difference is divisible by n.
4. Show that a number is divisible by 4 if and only if its last two digits are divisible by 4.
5. What is the last digit of 2013^{2013}? How about 2014^{2014}?

6. When Peter broke his piggy bank, it contained no more than 100 coins. He divided coins into piles of 2 coins each, but was left with one extra coin. The same happened when Peter divided the coins into piles of 3 coins, piles of 4 coins, and piles of 5 coins. Each time he was left with one extra coin. How many coins were in the piggy bank?

7. A set of numbers (a, b, c) with $a^2 + b^2 = c^2$ is called a *Pythagorean Triple.*

   (a) Show that in a Pythagorean triple, at least one of the numbers is divisible by 3.

   (b) Show that in a Pythagorean triple, at least one of the numbers is divisible by 5.