

UW Math Circle

April 17, 2014

Instructions: Kristin forgot what numbers are, so she decided to make up her own. Here are some numbers she invented:

123, 32154, 1423, 312, 2413, 162543

Steve decided to make up some numbers of his own, but Kristin told him that they don't count:

22, 243, 35, 541344, 1337

1. Which of these numbers count according to Kristin's definition of a number?

615324, 1327645, 31514, 12345, 541, 7, 41532, 3521, 312, 4132, 13542, 1, 14263675

2. (a) Write down all the one-digit numbers according to Kristin's rules. How many are there?

- (b) Write down all the two-digit numbers according to Kristin's rules. How many are there?

- (c) Write down all the three-digit numbers according to Kristin's rules. How many are there?

- (d) Write down all the four-digit numbers according to Kristin's rules. How many are there?

- (e) How many n -digit numbers are there according to Kristin's rules?

3. Next, Kristin made up rules for how to multiply two numbers together. Here are some of her results:

$$213 \cdot 321 = 231$$

$$2143 \cdot 4321 = 3412$$

$$21435 \cdot 54321 = 45231$$

$$214356 \cdot 654321 = 563421$$

$$2143567 \cdot 7654321 = 6745321$$

$$21435678 \cdot 87654321 = 78564321$$

$$321 \cdot 213 = 312$$

$$4321 \cdot 2413 = 3142$$

$$54321 \cdot 13524 = 42531$$

$$654321 \cdot 234651 = 156432$$

$$7654321 \cdot 2543671 = 1763452$$

$$1324 \cdot 3421 = 3241$$

$$1324 \cdot 3412 = 3142$$

$$1324 \cdot 3142 = 3412$$

$$1324 \cdot 1342 = 1432$$

$$1324 \cdot 1324 = 1234$$

$$1324 \cdot 1432 = 1342$$

$$123 \cdot 123 = 123$$

$$123 \cdot 321 = 321$$

$$123 \cdot 231 = 231$$

$$132 \cdot 231 = 213$$

$$213 \cdot 213 = 123$$

$$213 \cdot 321 = 231$$

$$213 \cdot 123 = 213$$

$$321 \cdot 321 = 123$$

$$231 \cdot 231 = 312$$

$$231 \cdot 213 = 132$$

$$23145 \cdot 41253 = 12453$$

$$3761542 \cdot 2617534 = 1432576$$

How would Kristin multiply these numbers:

$$312 \cdot 132 =$$

$$52143 \cdot 14235 =$$

$$21 \cdot 21 =$$

$$1347652 \cdot 3614527 =$$

$$4321 \cdot 2341 =$$

4. Can you find a number so that

$$31425 \cdot \underline{\hspace{2cm}} = 12345$$

$$\underline{\hspace{2cm}} \cdot 31425 = 12345$$

5. Start with a deck of 12 cards numbered one through twelve. Now deal them out into a 3×4 grid by laying them out along the rows from top to bottom:

1	2	3	4
5	6	7	8
9	10	11	12

Finally, collect the cards column-by-column from left to right. So now the top three cards on the deck are 1,5,9, followed by 2,6,10, and so on. How many times will you have to repeat this process before the cards are back in their original order?