

# UW Math Circle

April 3, 2014

Homework

1. Show that there are two different powers of 3 that differ by a multiple of 2014.
2. You have 10 identical coins, one of which is counterfeit, and hence slightly lighter than all the others. Show how to find the counterfeit coin using only 3 weighings on a balance scale.
3. A frog jumps along a straight line. Each time he jumps, he can move either 3 inches to the left or 5 inches to the right.
  - (a) Is it possible for him to move any number of inches to his right?
  - (b) Is it possible for him to move one inch to his right if he makes exactly 20 jumps?
  - (c) Is it possible for him to move 10 inches to his right if he makes exactly 23 jumps?
4. Greta the Gardner plants 2014 gooseberry shrubs haphazardly around the circumference of a circle. Prove that it is possible to find a diameter of the circle such that half of the shrubs are on either side.

5. Prove that if you can form a triangle with side lengths  $a$ ,  $b$ , and  $c$ , then you can also form a triangle with side lengths  $\sqrt{a}$ ,  $\sqrt{b}$ , and  $\sqrt{c}$ .

6. If  $ABCD$  is a convex quadrilateral and  $AB+BD < AC+CD$ , prove that  $AB < AC$ .

