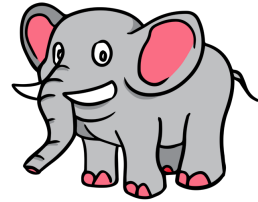


UW Math Circle

November 21, 2013

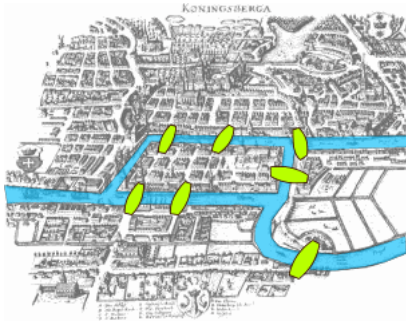
1. Jenny works at a zoo and is in charge of building habitats for the animals. She needs to build a habitat for anteaters, brown bears, cougars, deer, elephants, flamingos, giraffes, hippopotamuses, iguanas, and jaguars. But, some of the animals are more ... predatory than others. Therefore,
 - (i) Hippopotamuses cannot live with deer or giraffes.
 - (ii) Deer, flamingos, and anteaters all need separate habitats.
 - (iii) Iguanas can share their habitats only with jaguars.
 - (iv) Bears and elephants don't get along; neither do deer and elephants.
 - (v) Jaguars scare hippopotamuses, cougars, giraffes, and brown bears.
 - (vi) Deer and flamingos are also afraid of jaguars.

What is the fewest number of habitats Jenny could build?

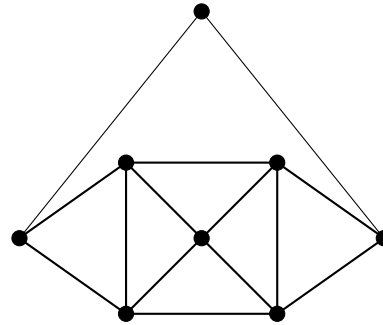
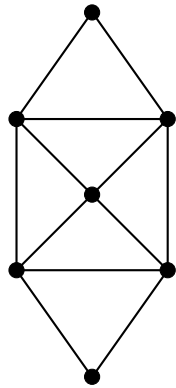
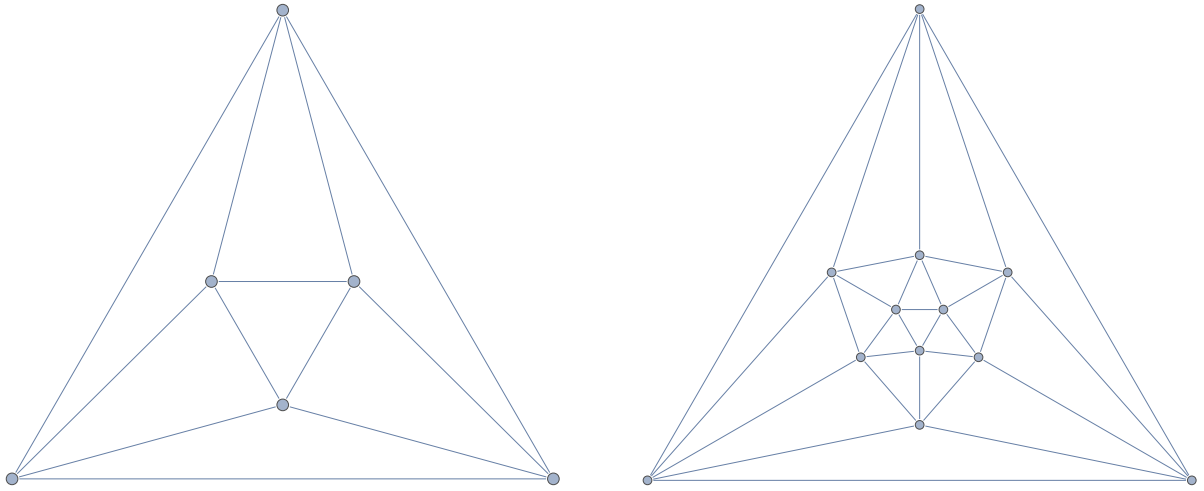


2. In the country of Septland, there are 15 towns. For each town, it is possible to fly directly (back and forth) between at least seven other towns. Show that it is possible to travel between any two towns in Septland by plane, provided that you can make transfer flights at some airports.

3. The picture below shows a map of the old city of Königsberg, in the principality formerly known as Prussia. As you can see, a system of rivers divides the city of Königsberg into four main land masses that are connected to one another by seven bridges. One day, Lenny decided that he wanted to make a grand tour of Königsberg and marvel in its spectacular bridges. Is it possible for Lenny to make a walking tour of Königsberg so that he crosses each bridge exactly one time?



4. Which of the following graphs can you draw without ever lifting your pencil from the paper? If you can trace the graph without lifting your pencil from the paper, can you also ensure that you start and end at the same vertex?



What do you notice? When can you trace a graph without lifting your pencil from the paper? In which cases can you draw the graph without lifting your pencil from the paper AND end at the same vertex where you started?