UW Math Circle
October 10, 2013
Worksheet: Week 3

THIS PROBLEM IS FROM LAST WEEK

Professor Snape is now ready to give you his hardest test! You meet him in a hallway with 9 closet doors, and he gives you the following information:

i Harry Potter is behind only one door.

ii For the other 8 rooms, each one either contains Voldemort or is empty.

iii The sign on the Harry Potter's room is true.

iv If Voldemort is behind the door, the sign on the door is false.

v If no one is behind the door, the sign can be either true or false.

After you've been working on the puzzle for a while, Professor Snape realizes he forgot to given you an important piece of information! "If I tell you what's behind door number 8, you'll be able to find Harry Potter."

I Harry Potter is in an odd-numbered room.	**II** This room is empty.	**III** Either sign V is right or sign VII is wrong.
IV Sign I is wrong.	**V** Either sign II is or sign IV is right.	**VI** Sign III is wrong.
VII Harry Potter is not in room I.	**VIII** Voldemort is in this room and room IX is empty.	**IX** Voldemort is in this room and sign VI is wrong.

Who is behind each door?

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1. Today is Simple Simon's birthday and he's decided to go to the fair. At the fair he decides to buy just a single piece of pie. Each piece of pie costs $2.00, and Simon pays in a combination of pennies, nickels, and quarters. Simon gives the pieman 33 coins for his piece of pie, and again, the pieman immediately says "Sorry, you can't add." How did the pieman know?

2. There is an island of chameleons, currently populated with 13 red chameleons, 17 blue ones, and 15 yellow ones. Whenever two chameleons meet, they change to the third color (so if a blue and a red chameleon meet, they become yellow). Is it possible for all the chameleons to become the same color?

3. The donkey is a new type of chess piece that can move either one square left/right and 4 squares up/down OR 3 squares left/right and one square up/down. There are two donkeys on an 8×8 chessboard – one in the upper left corner and another in the lower right corner. Every second, they both make a move. Is it possible that they could ever run into one another?

4. It's Studious Shoshana's birthday and you've been tasked with making two very big coconut cakes to celebrate. There are a total of 43 people at her party. You can cut each cake into either 3 or 5 pieces, and you can further cut each piece into either 3 or 5 pieces, and you can keep cutting pieces as long as you'd like. Is it possible to cut exactly 43 pieces of cake?

5. *Super Challenge.* Rachel is the leader of a committee of $6,234,634,893,456$ sentient beings from all around the universe. Everyone in this committee gets along with *almost* everyone else in the group, subject to the following rules:

   (a) Everyone has at *most* three other people that they don't get along with.
   (b) Everyone gets along with themselves.
   (c) If X doesn't get along with Y, then Y doesn't get along with X.

   Rachel figures out a way to divide the committee into two parts, A and B, so that everyone in committee A has at most one enemy in committee A and everyone in committee B has at most one enemy in committee B. How would you do it?