

UW Math Circle

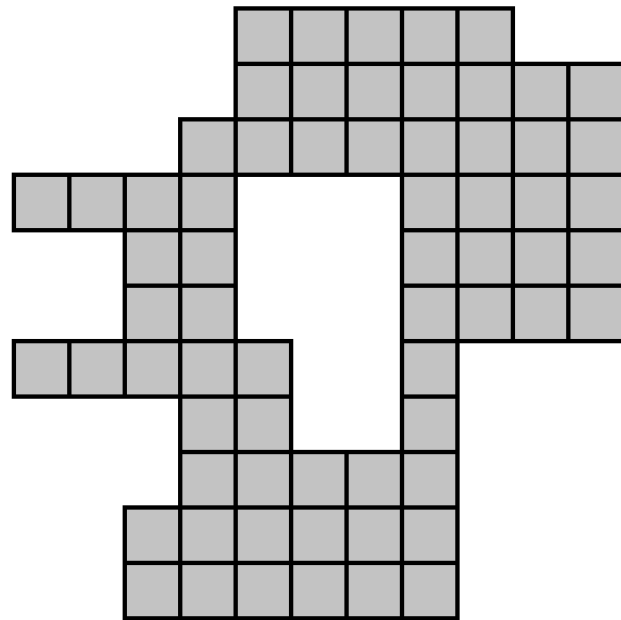
End-of-the-Year Auction

5 June 2014

Each problem is worth €100.

1. Place **as few lamps as possible** to light up all lattice points in the room shown here. Every lamp shines light in eight directions: N, NE, E, SE, S, SW, W, and NW. Assume that the walls act as mirrors and that light passes through corners.

2. Find **as many sets as possible** of 8 distinct positive integers such that their sum is divisible by each of the integers in the set. The greatest common divisor of all 8 numbers should be 1.
Example of such a set with 4 numbers: $\{1, 2, 3, 6\}$, but $\{5, 10, 15, 30\}$ does not count because all of the numbers are divisible by 5.



3. Place a rook, a knight, a bishop, a king, and a queen on an 8×8 chessboard so that **as many squares as possible** on the board are attacked. Assume that pieces do not attack the squares they stand on. Remember that pieces (except knights) cannot move through other pieces.

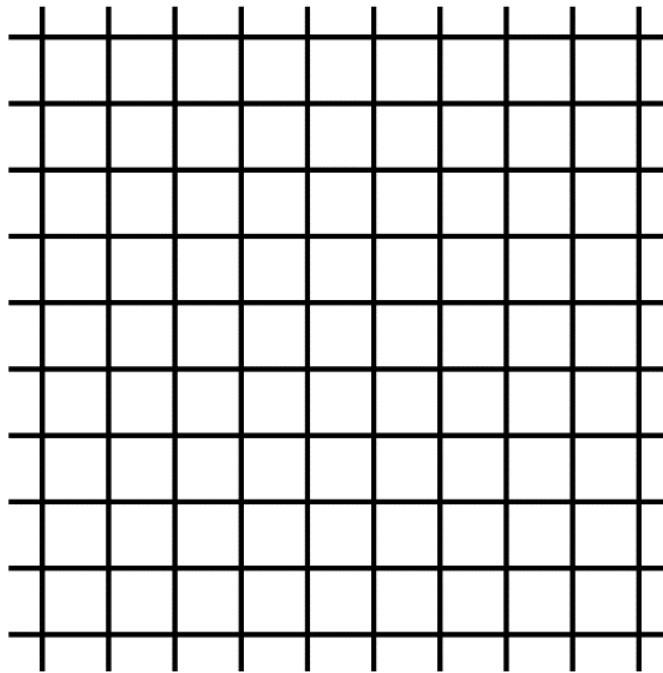
4. Place 16 letters in a 4×4 grid so that **as many distinct 4-letter words as possible** are spelt by reading left, right, up, down, and diagonally.
For example, this grid produces PAWS, SWAP, AREA, SNAP, PANS, SAWS, PASS, and SAPS for a total of 8 out of 20 possible words.

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P A W S
A R E A
S N A P
S A W S

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5. Color **as many lattice points as possible** in this 10×10 grid in such a way that no five of them are the vertices of a convex pentagon containing another of the colored points in its interior.



6. Consider the equality

$$ABC \times DE = FG \times HI.$$

Replace the letters with distinct digits from 1 to 9 to **make the product as large as possible**.

Example: 138×27=69×54. The product is 3726.