

Problem Set 9 Solutions

UW Math Circle – Advanced Group

Session 15 (30 January 2014)

1. See the official solution, <http://www.bamo.org/attachments/bamo2007examsol.pdf>.

2. (a) Yes with addition, no with multiplication (no inverses).
(b) Yes with addition, no with multiplication (0 has no inverse).
(c) No with addition (no identity), yes with multiplication.

3. Let $x, y \in G$. Setting $a = x^{-1}$ and $b = y^{-1}$, we get

$$\begin{aligned}(x^{-1}y^{-1})^{-1} &= (x^{-1})^{-1}(y^{-1})^{-1} \\ (y^{-1})^{-1}(x^{-1})^{-1} &= xy \\ yx &= xy.\end{aligned}$$

4. Suppose $(ab)^n = 1$. Then

$$(ba)^n = \underbrace{(ba)(ba)\dots(ba)}_n = b \underbrace{(ab)\dots(ab)}_{n-1} a = b \underbrace{(ab)\dots(ab)}_n b^{-1} = b(ab)^n b^{-1} = bb^{-1} = 1.$$

5. (a) Let G be a finite group and $a \in G$. Consider the sequence a, a^2, a^3, \dots . Because G is finite, there will be distinct m and n – suppose $m > n$ – such that $a^m = a^n$. But then $a^m a^{-n} = a^n a^{-n}$, or $a^{m-n} = 1$.
(b) Follows from (a).