Problem Set 8 Solutions

UW Math Circle – Advanced Group

Session 14 (23 January 2014)

1. See the official solution, `http://www.bamo.org/attachments/bamo2007examsol.pdf`.

2. (a) Show that you can dissect a square into 6, 7, or 8 squares, then show that you can dissect a square into 4 squares (adding 3 to the number of squares).

   (b) You can dissect a cube into 8 cubes (adding 7). You can also dissect a cube into 27 cubes (adding 26), so you can get 1, $1 + 26$, $1 + 2 \cdot 26$, ..., $1 + 6 * 26 = 157 < 200$. This is a complete set of remainders modulo 7. (We got this bound down to 78; the proven minimum is 48.)

3. (a) The 3×3 table is uniquely determined by the upper left 2×2 subtable. To see this, fill the upper left 2×2 subtable. We immediately know everything except the lower right corner. A parity argument shows the lower right corner is also uniquely determined. So, we just count the number of ways to fill the 2×2 subtable, which is $2^{2 \cdot 2} = 16$.

   (b) There are no ways to do this: suppose we filled a table with 3 rows and 4 columns in this way. Adding by rows, the sum of all numbers is odd. Adding by columns, it is even.

   (c) Similarly to (a), $2^{3 \cdot 3} = 512$. In general, for an $m \times n$ table, it is $2^{(m-1)(n-1)}$ if $m \equiv n \pmod 2$ and 0 otherwise.