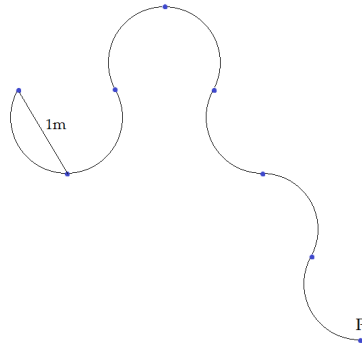# Problem Set 19

UW Math Circle – Advanced Group

Session 27 (15 May 2014)

1. Prove that any convex pentagon has three diagonals from which you can form a triangle.

2. (a) Roomba® moves in 120-degree arcs of circles, moving by 1 meter on each step, and makes no sharp turns. Roomba® starts at point P facing west. Could it ever end up at the point 1 meter west of P?

   (b) Same question if it moves in 90-degree arcs.
   (c) Same question if it moves in 60-degree arcs. Could it go to the point 1 meter north of P?

3. (MHO 2012) 120 bands are participating in this year's Northwest Grunge Rock Festival, and they have 119 fans in total. Each fan belongs to exactly one fan club. A fan club is called *crowded* if it has at least 15 members.

   Every morning, all the members of one of the crowded fan clubs start arguing over who loves their favorite band the most. As a result of the fighting, each of them leaves the club to join another club, but no two of them join the same one.

   Is it true that no matter how the clubs are originally arranged, all these arguments will eventually stop?