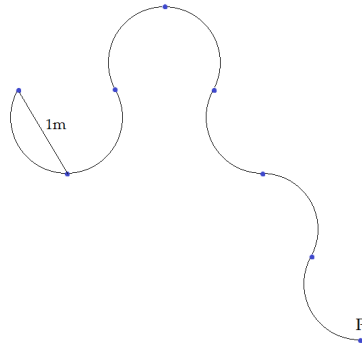


Problem Set 19

UW Math Circle – Advanced Group

Session 27 (15 May 2014)

1. Prove that any convex pentagon has three diagonals from which you can form a triangle.
2. (a) Roomba[®] moves in 120-degree arcs of circles, moving by 1 meter on each step, and makes no sharp turns. Roomba[®] starts at point P facing west. Could it ever end up at the point 1 meter west of P ?



- (b) Same question if it moves in 90-degree arcs.
 - (c) Same question if it moves in 60-degree arcs. Could it go to the point 1 meter north of P ?
3. (MHO 2012) 120 bands are participating in this year's Northwest Grunge Rock Festival, and they have 119 fans in total. Each fan belongs to exactly one fan club. A fan club is called *crowded* if it has at least 15 members.

Every morning, all the members of one of the crowded fan clubs start arguing over who loves their favorite band the most. As a result of the fighting, each of them leaves the club to join another club, but no two of them join the same one.

Is it true that no matter how the clubs are originally arranged, all these arguments will eventually stop?

An advertisement for "End of Proof Stickers". The ad features a grid of 90 small black squares arranged in a 10x9 grid. The text includes: "End of Proof Stickers", "Save time with these preshaded halmos stickers.", "Apply one sticker at the end of every completed proof.", "Easily combine four small stickers to create a large square for the end of particularly complicated proofs!", "May also be applied to your friend's face after winning an argument or debate using mathematics.", "Read More...", "add to wish list", "4,000,000 (save 99.999%)", "\$9.99 In stock", "Quantity: [1]", and a "BUY NOW" button. The website "spikedmath.com" and copyright "© 2013" are also visible.