Problem Set 17

UW Math Circle – Advanced Group

Session 24 (24 April 2014)

1. A permutation is called *123-avoiding* if it contains no three elements in increasing order. For example, [1 5 4 3 6 2] is not 123-avoiding because it contains the elements 1, 4, 6 (in that order).

   Here are the fourteen 123-avoiding permutations in S_4:

1432	2431	3241	4132	4312
2143	3142	3412	4213	4321
2413	3214	3421	4231	

   How many permutations in S_n are 123-avoiding? (Does it look familiar? Why? Bonus: Can you prove it using the Robinson-Schensted correspondence?)

2. A recently discovered manuscript written in Old Irish and titled "Táin bó agus cúnna" describes the following strange procedure:

   "The kings of Ulster, Leinster, Munster, and Connacht each had some bulls and some hounds. If a king saw that he had more hounds and fewer bulls than another, he could decide to steal a bull from him. If a king saw that he had fewer hounds and more bulls, he could steal a hound."

   Is it possible that these raids continued forever?

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3. Some red and blue lines are drawn in the plane. No two of them are parallel. Whenever two or more lines of one colour intersect, a line of the other colour must also go through the intersection point. Is it true that the lines must all intersect in a point?