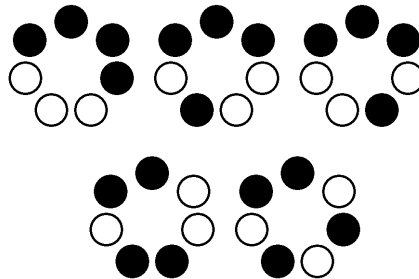# Problem Set 16

UW Math Circle – Advanced Group

Session 22 (3 April 2014)

1. Count the number of necklaces you can make with n black beads and $n-1$ white beads. Two necklaces are considered the same if you can get one from the other by rotating. Does it look familiar? (Why?) *Figure shows all necklaces for $n = 4$.*

2. Count the number of ways to tile an $n \times n$ "staircase" with exactly n rectangles. Does it look familiar? (Why?) *Figure shows all tilings for $n = 3$.*
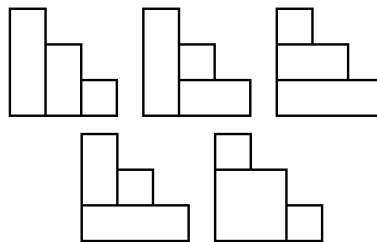
3. If G is a group, the *center* of G, denoted Z_G, is the set of elements that commute with all other elements of G (that is, $z \in Z_G$ if $zg = gz$ for all $g \in G$).

   (a) Prove that the center of a group G is a subgroup of G.

   (b) Prove that the center of the symmetric group S_n for $n > 2$ is the group with one element.