Problem Set 15

UW Math Circle – Advanced Group

Session 21 (27 March 2014)

1. An absent-minded mathematician wanders around the Mathematics Department.

   After spending a minute in his office, he goes with probability $\frac{1}{3}$ to the department lounge and with probability $\frac{2}{3}$ to the bus stop.

   After spending a minute at the bus stop, he goes with probability $\frac{2}{5}$ to his office and with probability $\frac{3}{5}$ to the department lounge.

   After spending a minute in the department lounge, he goes with probability $\frac{3}{4}$ to the bus stop and with probability $\frac{1}{4}$ to his office.

   After he has been doing this for a very long time, where should you look for him first? (And what chance do you have of finding him there?)

2. Checkers are placed in the three corner squares of an infinite board. In one move, you may remove one checker and place one checker each in the squares above and to the right, on the condition that these squares are empty. Is it possible to clear the three corner squares of checkers?
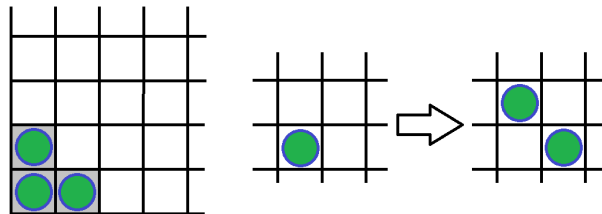
3. 7 rooks are placed on a 7×7 board so that no two rooks attack each other. Each rook makes one knight's move. Prove that now there are at least two rooks that attack each other.

4. Colour the plane in three colours in such a way that any line contains points of exactly two colours.