Problem Set 14

UW Math Circle – Advanced Group

Session 20 (20 March 2014)

1. 100 integers are written in a circle. The sum of all the integers is 1. An *arc* is a list of successive (written next to each other) numbers on the boundary. Find the number of arcs having a positive sum.

2. (USA 2008) At a certain mathematical conference, every pair of mathematicians are either friends or strangers. At mealtime, every participant eats in one of two large dining rooms. Each mathematician insists upon eating in a room which contains an even number of his or her friends. Prove that the number of ways that the mathematicians may be split between the two rooms is a power of 2.

   (Hint: This is just like the light switches problem. Start with one "good" seating and consider operations that move some subset of the mathematicians to the other room.)

3. Today we found a way to determine if a simply connected set of squares on a chessboard can be covered with domino tiles. Try to do the same thing for covering a region in the triangular lattice with "lozenges" (shown on the picture).