Problem Set 1

UW Math Circle – Advanced Group

Session 2 (3 October 2013)

1. Given a convex pancake, prove that you can cut it with a straight cut of a knife into two pieces of equal area and perimeter.

2. In a certain country there are many states. Each state is ruled either by the Tramecksan Party or the Slamecksan Party. Once a year, the ruling party in one of the states (A) may change. This occurs in the case that the majority of states neighboring A are ruled by a different party than the one that rules A. Prove that the changes in the governments of these states could not continue forever.

3. To celebrate space exploration, the Science Fiction Museum is going to read Star Wars and Star Trek stories for 24 hours straight. A different story will be read each hour for a total of 12 Star Wars stories and 12 Star Trek stories.

   George (a Star Wars fan) and Gene (a Star Trek fan) want to listen to exactly 6 Star Wars and 6 Star Trek stories. Show that no matter how the readings are scheduled, the friends can find a block of 12 consecutive hours to listen to the stories together.

4. A square field is divided into a 10×10 grid of small square plots. Weeds have infested 9 of the plots. The weeds will spread to a plot if two of the plots adjacent to it (i.e., sharing a side) are already infested. It is possible that the weeds will eventually take over the whole field? (Hint: What property of the region infested by weeds is invariant or semi-invariant when the weeds spread to a new plot?)

5. Several people are standing in a line. Some of them are facing left, some are facing right, and the rest are facing forward. Prove that I can find a place to stand in the line in such a way as to have an equal number of people facing me to my left and to my right.