# Problem Set 7

UW Math Circle – Advanced Group

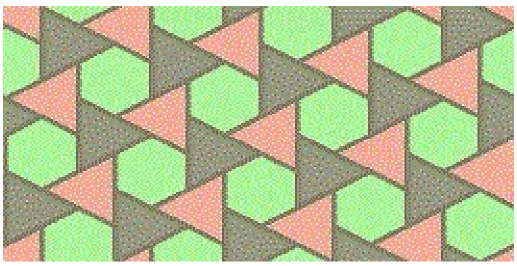
Session 10 (5 December 2013)

1. Use Bertrand's postulate / Chebyshëv's theorem to show that any positive integer can be written as a sum of distinct numbers that are prime or 1 (for example: $10 = 5 + 3 + 2$, $15 = 11 + 3 + 1$).

2. (a) (Euler's classic problem) Prove or disprove: $n^2 + n + 41$ is prime for all positive integers $n$.

   (b) (Goldbach, 1752) The goal of this problem is to show that there is no polynomial taking only prime values at positive integers.
   Suppose that $p(x) = x^n + c_{n-1}x^{n-1} + \cdots + c_2x^2 + c_1x + c_0$ is a polynomial with integer coefficients. Suppose also that $p(0), p(1), p(2), \ldots$ are all prime. Show that $p$ must be constant. *(Hint: Let $q = p(0)$ and consider $p(q), p(2q), p(3q), \ldots$[1].)*

3. Prove that you cannot fit more than 9 discs of diameter 1 in a $3 \times 3$ square without overlap.



[1] You may also want to use the fact that any nonconstant polynomial eventually goes off to $+\infty$ or $-\infty$ and cannot take on any value infinitely many times.