Math Circle - Introduction

1. Alex is extremely disorganized and keeps his socks all thrown together in a single drawer. He has 8 red socks, 10 blue socks, 4 green socks, and 4 white socks. In the morning, Alex rolls out of bed and blindly takes a handful of socks from the drawer. How many socks does he need to pull out to guarantee that he has at least one matching pair?
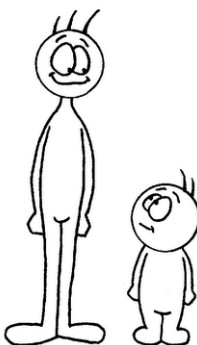
2. Chris lies a lot. He tells the truth on only one day in a week. One day he said: "I lie on Mondays and Tuesdays." The next day he said: "Today is either Sunday, Saturday or Thursday." The next day he said: "I lie on Fridays and Wednesdays." On which day of the week does Chris tell the truth?

3. Seven people have shown up to the latest Math Club party. The newest member suggests a game: each of the partygoers should try to shake hands with **exactly** three other Math Club members. Is this possible?

4. All the students in the math department at UW arrange themselves in a rectangular grid. The tallest student in each row was chosen, and then among these John Smith happened to be the shortest. After that, the shortest student was chosen in each column, and Mary Brown was the tallest of these. Who is taller: John or Mary?

5. Steve has four coins, one of which is counterfeit (either lighter or heavier than the other three, but he does not know which). Can Steve find the fake coin with just two weighings on a balance scale?

6. Kolya covers the exterior of a wooden cube with black paint. He then cuts the cube into 125 smaller cubes of exactly the same size. How many of the cubes have at least two of their sides painted black?

7. Wimpy Kid drew a 5×10 table and wrote a positive number in every cell so that the sum of the numbers in each of the five rows is equal, and also equal to the sum of the numbers in each of the ten columns. Wimpy Kid's mom is upset that he cannot count properly, but Wimpy Kid cries and says he did everything correctly. Who is right?

8. Three lines are drawn in the plane, dividing it into seven parts. Can you place the numbers from 1 to 7 in the seven parts so that the sums of the numbers on either side of each line are the same?