UW Math Circle - Homework 9

Recall that for a polynomial written as $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x_1 + a_0$ with roots $r_1, r_2, \ldots r_n$ we have

$$r_1 + r_2 + \ldots + r_n = -\frac{a_{n-1}}{a_n}$$
$$(r_1r_2 + r_1r_3 + \ldots + r_1r_n) + (r_2r_3 + r_2r_4 + \ldots + r_2r_n) + \ldots + r_{n-1}r_n = \frac{a_{n-2}}{a_n}$$
$$\vdots$$
$$r_1r_2r_3 \ldots r_n = (-1)^n \frac{a_0}{a_n}$$

these are known as **Vieta's formulas**.

We also learned the **Identity Theorem**: if two polynomials P_1 and P_2 of degree less than n take the same value at more than n points, then $P_1 = P_2$.

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1. Suppose the polynomial $5x^3 + 4x^2 - 8x + 6$ has three real roots a,b, and c. Find the value of $a(1 + b + c) + b(1 + a + c) + c(1 + a + b)$.

2. Suppose $P(x)$ is a polynomial such that $P(n) \geq 0$ for infinitely many positive integers n and $P(m) \leq 0$ for infinitely many positive integers m. Show that P is zero everywhere.

3. Find an x, y, and z such that

$$x + y + z = 17$$
$$xy + yz + xz = 94$$
$$xyz = 168$$

Hint: These equations look a lot like Vieta's Formulas - can you rephrase this problem in terms of polynomials?

4. A movie theater is holding a Batman marathon and is showing two Batman movies in a row. Fifty people show up to the first movie and the same fifty people show up to the second one. Prove that if the movie theater has 7 rows of seats, each with ten seats, then at least two people sat in the same row as each other for both movies.