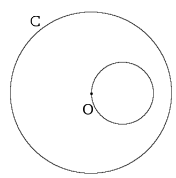
UW Math Circle - Homework 7

*Recall that an **inversion** with respect to a circle C centered at O with radius r is a map that takes any point P to a point P' that lies on the ray OP such that*

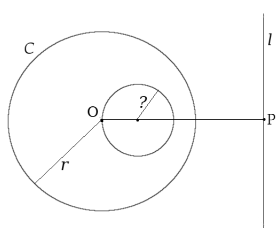
$$|OP| \cdot |OP'| = r^2$$



1. Suppose O is the center of a circle C with radius r. In class we proved that an inversion with respect to C takes circles that do *not* pass through O to circles. Prove (in a very similar fashion!) that the same inversion takes lines that do not pass through O to circles that *do* pass through O. Notice that we now know what happens to *all* circles under inversion: if the circle passes through O it is mapped to a line, otherwise it is mapped to a circle that does not pass through O.

2. What does the inverse of a triangle look like? Make sure you consider all cases!

3. Suppose O is the center of a circle C with radius r and l is a line that does not pass through O. Draw a line from O perpendicular to l and let P be the point where this line intersects l. Find the radius of the circle you get when you invert l with respect to C. Write this radius in terms of $|OP|$ and r. What if l *does* pass through O? What does your radius become? What does this mean?



4. Suppose O is the center of a circle C with radius r and D is a circle with center A that does not pass through O. Is the inverse of A with respect to C the center of the inverse of D with respect to C? In other words: is the inverse of the center the center of the inverse?