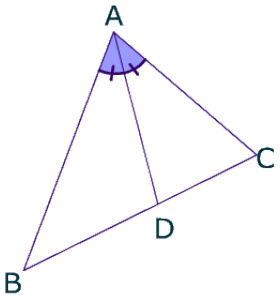
UW Math Circle - Homework 6

1. In class we showed how to, given a line segment AB and a line segment of length 1, divide AB into m equal pieces for any $m > 1$. The first step of our construction was to "draw any line through A that is not parallel to B." Find a faster and easier construction by starting with drawing a line perpendicular to AB at A. Make sure to prove that your construction is correct!

2. In class we showed how to, given a line segment AB and a line segment of length 1, construct a line segment of length $\sqrt{AB}$. Given any line segment AB and a line segment CD, show how to construct a line segment of length $\sqrt{AB \cdot CD}$



3. Given a triangle ABC and an angle bisector AD as shown in the figure, show that

$$\frac{BD}{DC} = \frac{AB}{AC}$$

4. *Challenge:* State and prove a similar theorem as in problem 3, but for the bisectors of exterior angles (in the figure, the exterior angle of $\angle BAC$ is $\angle BAO$, and AE is the bisector of $\angle BAO$).