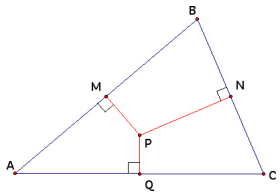
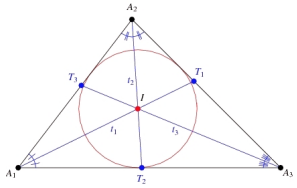
UW Math Circle - Homework 5.5

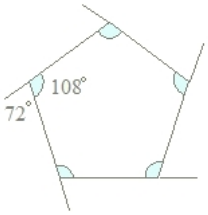
Last week we spent our time preparing for the Bay Area Mathematical Olympiad (BAMO). The UW Math Circle strongly encourages everyone participating in the competition to look at previous exams and solutions to get an idea of kinds of problems to expect. Although we did not cover any new geometry material this week, below are some problems that you can think about if you are done with Homework 5. We will discuss both Homework 5 and Homework 5.5 next week.*

1. Prove that the three perpendicular bisectors of a triangle intersect at one point, and this point is the center of the circumscribed circle of the triangle. Similarly, prove that the three angle bisectors of a triangle intersect at one point, and this point is the center of the inscribed circle of the triangle.





2. We have shown how to construct angles of 90° and 60°, and by constructing angle bisectors we can build angles of $45°, 22.5°, 11.25°, \ldots$ and $30°, 15°, 7.5°, \ldots$ and any sum or difference of these. Show how to construct an angle of 72°. Notice that 72° is the measure of the exterior angles of a regular pentagon, so this problem is equivalent to constructing a regular pentagon (why?).



*You can find previous BAMO exams at `www.bamo.org/archives`