UW Math Circle - Homework 3

1. Find the gcd of (123456789, 41234595) and of (75342124, 154312346) without using a calculator. Write each gcd as a linear combination of the two numbers.

2. Aquaman has a bucket that holds 5 liters, a bucket that holds 11 liters, and a superpower that lets him *completely* fill or *completely* empty either bucket of water. Aquaman can also pour water from one bucket into another. How can Aquaman use these buckets and his superpower to measure out exactly 5 liters of water? Note: there are no measuring lines on the buckets! Aquaman cannot, for example, pour exactly half of the water in one bucket into the second bucket.

3. Prove that if $a|bc$ and $(a, b) = 1$ then $a|c$. (**Hint:** Write 1 as a linear combination of a and b, and multiply that combination by c).

4. Steve and José are playing the following game: José picks two numbers between 1 and 100 and then Steve does Euclid's algorithm to José's numbers. José gets a point every time that Steve has to divide two numbers to get a smaller residue - what numbers should he pick to get the maximum number of points? How many points will José earn if he chooses this pair of numbers? Which numbers should he pick if he can choose any numbers between 1 and 1000?

5. Alex has a bag with 1000 pieces of paper. For each number k between 1 and 1000 there is one paper that says: "at least k of the papers in the bag are lying." How many papers are telling the truth?