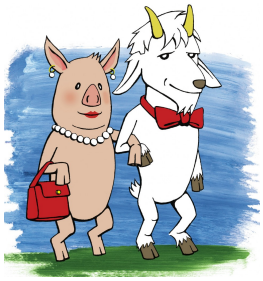
UW Math Circle - Homework 2

1. (a) In a special football game, a team scores 7 points for a touchdown and 3 points for a field goal. Determine the largest mathematically unreachable number of points scored by a team in a game. Why is it that there exists that largest number?

   (b) Two farmers agree that pigs are worth \$300 and goats are worth \$210. When one farmer owes the other money, he pays the debt in pigs or goats, with change received in the form of goats or pigs as necessary. (For example, a \$390 debt could be paid with pigs, with one goat received in change). What is the amount of the smallest positive debt that can be resolved in this way?



2. On Mafia Island there are only two types of coins - one is worth \$7 and the other is worth \$11. Mafia Island has no banks, so everything must be paid in cash. José owes three mob bosses different amounts of money: to one he owes \$194, to another \$26, and to the last \$3. It is very embarrassing to ask mob bosses for change, which of these bills can José pay using \$7 and \$11 coins? What kind of change will José have to ask for if he wants it to involve the smallest possible amount of coins? Can Jos pay any quantity he wants?

3. Look at the last two digits of every number in the sequence

$$1, 7, 7^2, 7^3, \ldots$$

Show the remaining sequence is *periodic* - it has a pattern that repeats over and over. How many unique numbers are there in this pattern?

4. Show that if $a|b-c$ and $a|d-e$ then $a|bd-ce$. (**Hint:** try to get a bd in the expression on the left and a $-ce$ in the expression on the right. Use the big fact we proved last week: if $a|b$ and $a|c$, then $a|xb+yc$ for all integers x and y)

5. The numbers 1, 2, 3, ..., 2013 are written on a board. Alex plays a game where he erases two numbers at each step and writes their (positive) difference until only one number is left on the board. Is the last number written on the board even or odd? What is the largest possible value of the last number on the board?